

# Guide to Curve Sketching

A Summary of What We Can Learn About the Graph of a Function

## Domain and Range

Domain: Are there discontinuities in  $f(x)$ ?

A rational function where the denominator = 0

Radical function (even root) where radicand  $< 0$

Range: What is the maximum and/or minimum of  $f(x)$ ?

Where are the tails pointing? Look at  $x \rightarrow \infty^\pm$ . What happens to  $f(x)$ ?

Note that polynomials of even degree have their tails pointing in the same direction so there will be a maximum or minimum for the function.

Other infinities created at asymptotes?

## Intercepts

y-intercept: A function has, at most, one. Evaluate:  $f(0) = ?$

x-intercept  $f(x) = 0$ , solve for  $x$ .

a) Solve analytically

b) Solve numerically by using graphing calculator or Intermediate Value Theorem to estimate root(s). Then, use Newton's Method to find a precise value.

## Continuity

Is the function continuous for all points,  $c$ , in the domain? There are three conditions:

a)  $f(c)$  must be defined.

b)  $\lim_{x \rightarrow c} f(x)$  exists.

c)  $\lim_{x \rightarrow c} f(x) = f(c)$

## Vertical Asymptotes

a) Reduce a rational expression to lowest terms. (Common factors between numerator and denominator produce removable, one-point discontinuities.)

a) Set *reduced* denominator = 0 and solve.

## Horizontal Asymptotes

$f(x)$  is asymptotic to the horizontal line  $y = L$  (where  $L$  is some constant, real number) if:

a)  $\lim_{x \rightarrow \infty^+} f(x) = L$  or b)  $\lim_{x \rightarrow \infty^-} f(x) = L$

(Note that the limit on the left does not have to equal the limit on the right.)

If the degree of the numerator = 1 + degree denominator, the oblique asymptote is the line

$$q(x) \text{ where } \frac{n(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}.$$

## Relative Extrema

Relative extrema occur at the function's critical number(s):

End points. (see Domain)

Where  $f'(x)$  does not exist.

Values of  $x$  where  $f'(x) = 0$ .

Remember, the extrema are *points* with coordinates  $(x, f(x))$ .

## Concavity

Use the second derivative to determine which way the graph is curving:

$f''(x) > 0$  means the graph is concave up.

$f''(x) < 0$  means the graph is concave down.

## Relative Maxima and Minima

Relative maxima and minima occur at the critical numbers ( $c.n.$ ) and can be identified by the

2<sup>nd</sup> Derivative Test:

$f''(c.n.) > 0$ , the point is a relative minimum.

$f''(c.n.) < 0$ , the point is a relative maximum.

## Points of Inflection

$f''(x) = 0$  and solve for  $x$ .

Also where  $f''(x)$  does not exist.

Remember inflection points are *points* with coordinates  $(x, f(x))$ .

## Limits at Infinity

Where are the “tails” of the graph pointing? (See Range)

If  $x \rightarrow \infty^{\pm}$ ,  $f(x) \rightarrow \infty^{\pm}$  ?

$f(x) \rightarrow L$  is a horizontal asymptote.